PHY801: Survey of Atomic and Condensed Matter Physics
Michigan State University

Homework 1

1.1. Using a hydrogenic model, estimate the 1st ionization energy of a Li atom, assuming that the two electrons in the 1s state essentially screen the nuclear charge, thus making its effective charge $+1e$. The observed value of the 1st ionization energy is 5.39 eV. Discuss possible physical reasons for the difference between the estimated and the observed value.

1.2. Calculate the 3rd ionization energy of the Li atom. Is your answer exact?

1.3. What is the probability of finding the $1s$ electron in Pb^{81+} inside the Pb nucleus? Assume that the nuclear radius $R = r_0 A^{1/3}$, where $r_0 = 1.2$ fermi and A is the atomic mass number (which differs from the atomic number Z!) of Pb.

1.4. Excitons in quantum wells and their binding energies can be approximated by a 2-dimensional (2D) hydrogen atom model. To use this description, first separate the radial part $R(r)$ and the angular part $Y(\theta)$ of the wavefunctions in the Schrödinger equation. Show that the radial part of the wavefunctions is the solution of (in atomic units)

$$\frac{1}{2}(R'' + \frac{1}{r}R') - \frac{m^2}{2r^2}R + (E + \frac{1}{r})R = 0 \ .$$

The angular part of the wave functions is given by $e^{im\theta}$. R' is the first and R" the second derivative of $R(r)$ with respect to r. Use the same scaling that was used in the 3D case in defining the variable $\rho = \kappa r$. Use $\kappa = (-2E)^{1/2}$ when writing down the second order differential equation for $R(\rho)$ in terms of the parameter $\rho_0 = 2/\kappa$. How does $R(\rho)$ behave as $\rho \to 0$ and $\rho \to \infty$? Define a function $v(\rho)$ following the same procedure as in the 3D case. Solve this equation and identify physical solutions, which provide the spectrum of the 2D hydrogen atom.
