PHY801: Survey of Atomic and Condensed Matter Physics
Michigan State University

Homework 7

7.1. Use the following equation for the drift velocity of an electron in a constant electric field oscillating with angular frequency ω,

$$m\left(\frac{dv}{dt}+\frac{v}{\tau}\right)=-eE(t)$$

to show that the frequency-dependent conductivity (*ac* conductivity) is given by

$$\sigma(\omega)=\sigma(0)\frac{1+i\omega\tau}{1+(\omega\tau)^2}\ ,$$

where $\sigma(0)=\frac{ne^2\tau}{m}$ and n is the density of electrons.

7.2. (a) Consider a square lattice with lattice constant a. Draw the 1st BZ and give the coordinates of it's symmetry points (use 4-fold rotation about the z-axis here). Now consider the states of an electron, whose $\vec{k}$ vector corresponds to one of the corner points of the BZ. How many plane waves have the same kinetic energy as the one corresponding to this $\vec{k}$? What are their $\vec{k}$ vectors? (We will call these as degenerate set of plane waves, because in the absence of a crystal potential these plane waves have the same kinetic energy.)

(b) Construct two stationary wave functions $\psi_1(x,y)$ and $\psi_2(x,y)$ from the above degenerate set such that for one [$\psi_1(x,y)$] the probability of finding the electron at the lattice sites is maximum and for the other [$\psi_2(x,y)$] it is minimum.

7.3. Now consider the case when there is a periodic potential in this square lattice, given by

$$U(x,y)=-U_0\left[\cos\left(\frac{2\pi}{a}x\right)+\cos\left(\frac{2\pi}{a}y\right)\right]\ .$$

What is the energy gap between the two states $\psi_1(x,y)$ and $\psi_2(x,y)$, defined in Problem 7.2.? Explain your result physically by matching the probability distribution of the electron in these two states and the attractive regions of the potential.
