PHY801: Survey of Atomic and Condensed Matter Physics
Michigan State University, Spring Semester 2020

Practice Exam 1

Student Name:

Useful Formulas:

- **Magnetization** M of a multiplet with a given total angular momentum J and Landé $g-$factor g_J is given by

$$\begin{aligned} M &= N g_J \mu_B J B_J(x) \text{ , where} \\ x &= \frac{g_J \mu_B J B}{k_B T} \text{ and} \\ \mu_B &= \frac{e\hbar}{2m} = 5.8\times 10^{-5} \text{ eV/T} \quad \text{is the Bohr magneton.} \end{aligned}$$

The Brillouin function is given by

$$B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{(2J+1)x}{2J}\right) - \frac{1}{2J}\coth\left(\frac{x}{2J}\right) .$$

It may be approximated by

$$B_J(x) \approx \frac{J+1}{3J} x \text{ for } x << 1 .$$

- **Hydrogen atom (Z=1)**:

$$\begin{aligned} R &= \frac{me^4}{2\hbar^2} = 13.6 \text{ eV} \quad \text{Rydberg constant} \\ a_B &= \frac{\hbar^2}{me^2} = 0.529 \text{ Å} \quad \text{Bohr radius} \end{aligned}$$

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1. (5 points) X-ray laser pulses can be used to strip atoms of electrons one by one (ionize the atom).

(a) What is the energy required to remove the last electron from a K atom? The neutral K atom has 19 electrons. The ionization energy of the hydrogen atom is 13.6 eV.

(b) What is the size of the Bohr radius for this last electron?

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2. (5 points) Consider an electron trapped in a 3-dimensional harmonic oscillator potential. The Hamiltonian in atomic units ($\hbar = m = e^2 = 1$) is given by

$$H = -\frac{1}{2}\nabla^2 + \frac{1}{2}r^2 \, .$$

Let's use a trial wave function for the ground state of the form

$$\psi(\vec{r}) = \frac{1}{\sqrt{\pi\alpha^3}} e^{-r/\alpha} \, .$$

This gives (do **not** derive this)

$$\frac{<\psi|H|\psi>}{<\psi|\psi>} = \frac{1}{2\alpha^2} + \frac{3}{2}\alpha^2 \, .$$

What is the best variational ground state energy one can get (in Hartree units)?

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3. (10 points) The ground state electronic configuration of the nitrogen atom is $1s^2\ 2s^2\ 2p^3$. Treat the $1s$ and $2s$ electrons as core electrons and the $2p$ electrons as valence electrons.

(a) What is the total spin angular momentum and total orbital angular momentum of the core? Do these core electrons contribute to the magnetic susceptibility χ of the atoms? If so, what is the sign of χ?

(b) According to Hund's rules, what is the lowest energy multiplet and what is its degeneracy?

(c) What is the expression for zero-field magnetic susceptibility of 1 mole of nitrogen atoms at high temperatures T?

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4. (5 points) A He atom is excited to the $1s^1\ 2p^1$ configuration. The one-electron spatial wave functions are $\phi_{1s}(\vec{r})$ and $\phi_{2p_m}(\vec{r})$.

(a) What is the total number of 2-electron states corresponding to this configuration?

(b) Write down the 2-electron spin triplet excited state wave functions associated with this configuration.

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5. (5 points)

(a) Write down the wave function for the ground state of the H_2 molecule in the molecular orbital (MO) approximation. Express your answer in terms of the $1s$ hydrogen atom wave functions associated with the two protons, $a(\vec{r})$ and $b(\vec{r})$.

(b) Draw qualitatively the ground state electronic energy, the repulsive energy between two protons, and the total energy as function of the inter-nuclear separation R.
